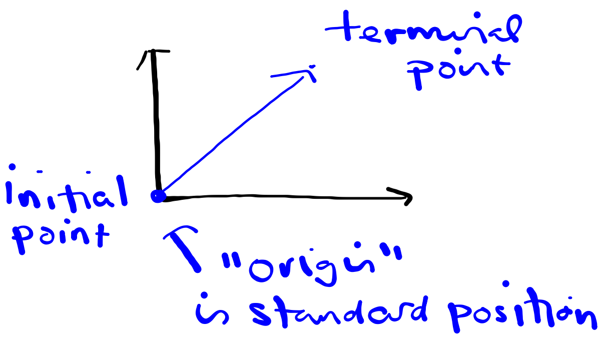
6.1 Vectors in the plane

Scalars are quantities that have magnitude (size) only.
e.g. speed, distance, temperature

Vectors are quantities that have magnitude and direction.
e.g. velocity, displacement, force

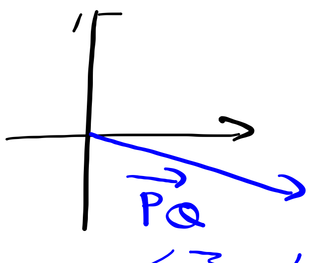
A vector is a directed line segment.



bold italic

$\underline{v}$ $\mathbf{v}$ v $\vec{v}$

$\overrightarrow{AB}$



$\langle 3, -4 \rangle$

component form

Intro to Calc

Equivalent vectors

Two vectors that represent the same vector are equivalent.
The vector represented by initial point (x_1,y_1) and terminal point (x_2,y_2) is $\underline{v}$ $<x_2-x_1,\ y_2-y_1>$

Ex. 1
Given A=(-4,2), B=(-1,6), C=(2,-1) and D=(5,3), are vectors $\vec{AB}$ and $\vec{CD}$ equivalent?

$\vec{AB} = \langle -1-(-4),\ 6-2 \rangle$
$= \underline{\langle 3, 4 \rangle}$

$\vec{CD} = \langle 5-2,\ 3-(-1) \rangle$
$= \underline{\langle 3, 4 \rangle}$

So $\vec{AB}$ and $\vec{CD}$ are equivalent.

$\vec{DC} = \langle 2-5,\ -1-3 \rangle$
$= \langle -3, -4 \rangle$

$\vec{DC}$ and $\vec{AB}$ are not equivalent.

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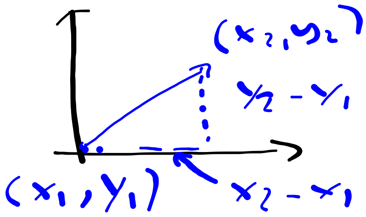
Magnitude

If **v** is represented by the arrow from

(x_1,y_1) to (x_2,y_2) then

If **v**<a,b> then

$$|\underline{v}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



Ex. 2

Find the magnitude of **v** represented by $\overrightarrow{AB}$

where A=(-3,4) and B=(-5,2) .

$$|\underline{v}| = \sqrt{(-5-(-3))^2+(2-4)^2}$$

$$= \sqrt{8}$$

$$= \underline{\underline{2\sqrt{2}}}$$

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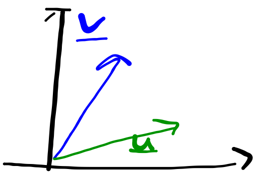
Vector operations

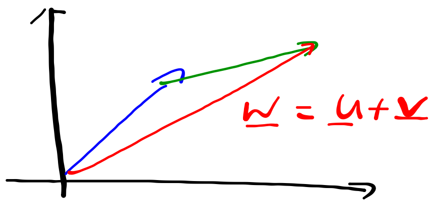
Scalar multiplication k<a,b>=<ka,kb>

$3\langle 1,-3\rangle = \langle 3,-9\rangle$

If **u**=<x_1,y_1> and **v**=<x_2,y_2> then the sum or resultant vector **u**+**v**=<x_1+x_2,y_1+y_2>

Graphically:





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Ex.3

Let $\mathbf{u}=\langle 4,-5\rangle$ and $\mathbf{v}=\langle 5,8\rangle$, find:

(a) $\mathbf{u}+\mathbf{v}$

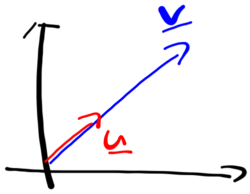
(b) $5\mathbf{v}$

(c) $3\mathbf{u}-4\mathbf{v}$

Unit Vectors

A unit vector **u** has length |**u**| =1. If **v** is not the zero vector, then

$$\underline{u} = \frac{\underline{v}}{|\underline{v}|}$$



is a unit vector in the direction of **v**.

Ex.4
Find a unit vector in the direction of **v**<4,-7>, and verify that it has length 1.

$$\underline{u} = \frac{\underline{v}}{|\underline{v}|}$$

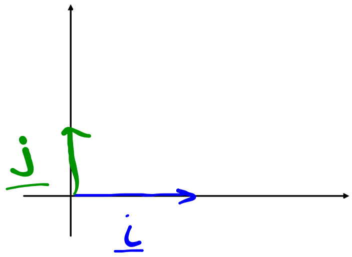
$$\underline{v} = \sqrt{4^2 + (-7)^2}$$
$$= \sqrt{65}$$

$$\underline{u} = \frac{\langle 4, -7 \rangle}{\sqrt{65}}$$
$$= \left\langle \frac{4}{\sqrt{65}}, -\frac{7}{\sqrt{65}} \right\rangle$$
$$= \left\langle \frac{4\sqrt{65}}{65}, -\frac{7\sqrt{65}}{65} \right\rangle$$

$$|\underline{u}| = \sqrt{\frac{4^2}{\sqrt{65}} + \left(\frac{-7}{\sqrt{65}}\right)^2}$$
$$= \sqrt{\frac{16}{65} + \frac{49}{65}}$$
$$= \sqrt{\frac{65}{65}}$$
$$= \underline{1}$$

Standard Unit Vectors

i=<1,0>
j=<0,1>



component form
$\langle 2, -3 \rangle$
$2i - 3j$
linear combination

v=<a,b>=a**i**+b**j** – linear combination of vectors **i** and **j**.
a – the horizontal component
b – the vertical component

Ex.5 Find a unit vector in the direction **v**=<5,-3>. Write your answer in both component form and as a linear combination.

$$\underline{u} = \frac{\underline{v}}{|\underline{v}|}$$

$$|\underline{v}| = \sqrt{5^2+(-3)^2} = \sqrt{34}$$

$$\underline{u} = \frac{\langle 5,-3 \rangle}{\sqrt{34}}$$

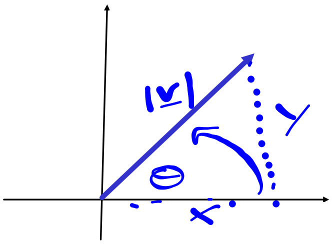
$$\underline{u} = \left\langle \frac{5\sqrt{34}}{34}, -\frac{3\sqrt{34}}{34} \right\rangle$$ component form

$$\underline{u} = \frac{5\sqrt{34}}{34} i - \frac{3\sqrt{34}}{34} j$$ linear combo

Intro to Calc

Direction angles

The direction angle θ is measured from the positive x-axis.



$$\frac{x}{|v|} = \cos\theta$$

$$\frac{y}{|v|} = \sin\theta$$

Splitting the components into

$x = |v|\cos\theta$

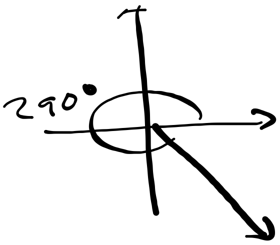
$y = |v|\sin\theta$

$\langle |v|\cos\theta, |v|\sin\theta \rangle$

is called resolving the vector.

Ex. 6

With the aid of a sketch, find the components of **v** with direction angle 290° and magnitude 4.



$\underline{v} = \langle 4\cos 290°, 4\sin 290° \rangle$

$\underline{v} = \langle 1.368, -3.759 \rangle$

Intro to Calc

Direction Angle

The direction angle, θ, can be found in two ways.

1) Equate the 'x' or 'y' coordinate with the definition for x or y and solve for θ.

OR

2) Use the following formula: $\tan\theta = \dfrac{y}{x}$

Ex. 7

Find the magnitude and direction angle of

(a) **u**=<1,4> (b) **v**=<-5,-4> (c) **w**=<3,-2>

(a)

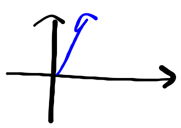
$u = \sqrt{1^2+4^2}$

$= \sqrt{17}$

$\tan\theta = \dfrac{4}{1}$

$\theta = \tan^{-1} 4$

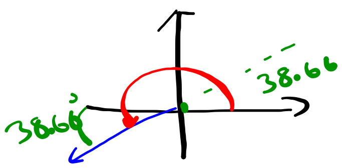
$\theta \approx 79.564°$



(b)

$|v| = \sqrt{(-5)^2+(4)^2}$

$= \sqrt{41}$



$\tan\theta = \dfrac{-4}{-5}$

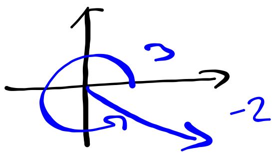
$\theta_r = \tan \dfrac{-4}{-5}$

$\theta_r \approx 38.66°$

$\theta \approx 180 + 38.66$

$\approx 218.66°$

(c)



$|w| = \sqrt{3^2+(-2)^2}$

$= \sqrt{13}$

$\theta_r = \tan^{-1} \dfrac{-2}{3}$

$\theta_r \approx -33.692°$

$\theta = 360 - 33.692$

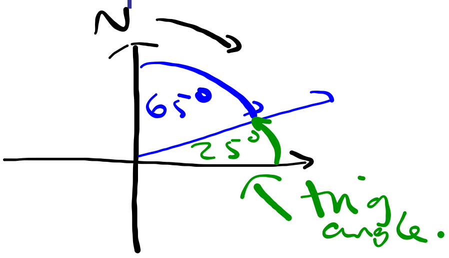
$\approx 326.31°$

Intro to Calc

Velocity is a vector, the magnitude of velocity is a scalar quantity and it is called speed.

Ex. 8

A jet aircraft is flying at a bearing of 65° at 500mph. Find the component form of the velocity of the airplane.



$\underline{P}$ = plane vector

$\underline{P} = \langle 500\cos 25°, 500\sin 25° \rangle$

$\underline{P} = \langle 453.154, 211.309 \rangle$

453.154 mph is the horizontal speed.

211.309 mph is the vertical speed.

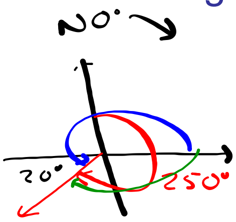
Ex. 9

Mr. George is flying on a bearing of 250° at 370mph. there is a wind blowing with a bearing of 300° at 75mph.

(a) Find the component form of the velocity of the airplane.

(b) Find the actual ground speed and ~~direction~~ bearing of the plane.

a)



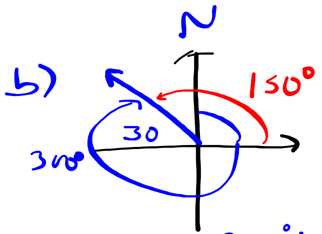
250° bearing ⟷ 200° trig

− 160°

$\underline{p} = \langle 370\cos 200°, 370\sin 200° \rangle$

$\underline{p} = \langle -347.686, -126.547 \rangle$

b)



300° bear ⟷ 150° trig

$\underline{w} = \langle 75\cos 150°, 75\sin 150° \rangle$

$= \langle -64.952, 37.5 \rangle$

$\underline{gs} = \underline{p} + \underline{w}$

$\underline{gs} = \langle -412.638, -89.047 \rangle$

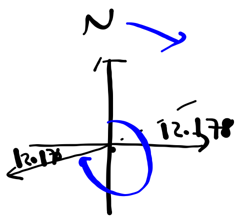
$|\underline{gs}| = \sqrt{(-412.638)^2 + (-89.047)^2}$

$|\underline{gs}| \approx \underline{422.137}$

$\tan\theta = \dfrac{-89.047}{-412.638}$

$\theta_r \approx \tan^{-1}\left(\dfrac{-89.047}{-412.638}\right)$

$\theta_r \approx \underline{12.178°}$



$\theta \approx 270 - 12.178°$

$\approx \underline{257.822°}$

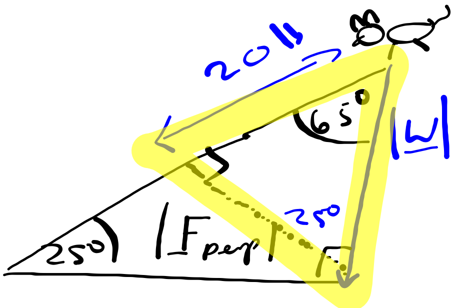
Ground speed $\underline{gs}$ ≈ 422 mph and bearing ≈ $\underline{257.822}$

Ex. 10

A force of 20lb keeps Bob, a Labrador, from sliding down an icy hill, which has an incline of 25°.

(a) How heavy is Bob?

(b) What is the force perpendicular to the icy hill?



$|w|$ = Bob's weight

$|F_{perp}|$ = ⊥'r force.

$$\frac{20}{|w|} = \sin 25°$$

$$20 = |w| \sin 25°$$

$$\frac{20}{\sin 25°} = |w|$$

$$|w| \approx \underline{47.32 \text{lb}}$$

$$\frac{|F_{perp}|}{20} = \tan 65°$$

$$|F_{perp}| = 20 \tan 65°$$

$$\approx \underline{42.9 \text{lb}}$$